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Smoothed Particle Hydrodynamics – A Meshless Particle Method

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Introduction:

- Recently, there has been a growing interest in researchers for developing meshfree particle methods (MPM) due to their advantages over the conventional grid-based methods for many applications.
- MPMs do not require a fixed connectivity between nodes as in grid based methods, thus making treatment of large deformations much easier.
- Smoothed particle hydrodynamics is a meshless particle method, which has been widely used for solving heat transfer and fluid dynamic problems in the field of engineering..
- It has an attractive feature that the interpolation points has mass and they act as material component by carrying the properties of the material along with them as they move according to internal and external interactions.
- In SPH, a smoothing kernel function is used to approximate the field variables and its derivatives at a particle from its neighboring particles and to solve the governing partial differential equations.
- In this work, two computational models have been developed based on smoothed particle hydrodynamics to solve steady state heat conduction problems and classical fluid dynamic problems respectively.
- Through numerical simulations, it is found that the results obtained from our developed model matches very well with available results in the literature.
- We believe that this model can be easily extended to solve complex fluid dynamic problems that involve fluid-structure interactions..

Literature:

- Smoothed particle hydrodynamics (SPH) was originally invented for modeling astrophysical problems in three-dimensional open space [1].
- The conventional SPH has a low accuracy and it cannot exactly reproduce even a constant function due to particle inconsistency [2].
- Several approaches have been proposed to restore the particle consistency.
- One way is reconstructing the smoothing kernel to satisfy discretized consistency conditions as in reproduced kernel particle method (RKPM) proposed by Liu et al. [3].
- Another way is to construct improved SPH approximation schemes using Taylor series expansions. Corrective smoothed particle method (CSPM) proposed by Chen et al. [2], finite particle hydrodynamics (FPM) proposed by Liu et al. [4] and decoupled finite particle method (DFPM) proposed by Zhang and Liu [5] are schemes using Taylor series expansions.
- The CSPM and FPM schemes calculates the field variables and its derivatives by inverting a matrix, which may lead to numerical instability or unexpected termination of simulation for highly disordered particle distributions.
- In DFPM, there is no need to solve matrix equations thus it is flexible, stable and computationally more efficient.

Methods:

- The field variables at a particle are calculated from its neighboring particles using a kernel function.

$$< f(\mathbf{x}_i) >= \sum_{j=1}^{N}\frac{m_j}{\rho_j} f(\mathbf{x}_j)\cdot W_{ij} \qquad (1)$$

$$< \nabla\cdot f(\mathbf{x}_i) >= \sum_{j=1}^{N}\frac{m_j}{\rho_j} f(\mathbf{x}_j)\cdot \nabla_i W_{ij} \qquad (2)$$

- where i and j represents each particles, N is the number of neighboring particles in the support domain of particle i, m is the mass, ρ is the density, $W_{ij} = W(x - x', h)$ is the smoothing kernel function, $\nabla_i W_{ij} = \frac{x_j - x_i}{r_{ij}}\frac{\partial W_{ij}}{\partial r_{ij}}$ and x is the position vector. Here, h is the smoothing length that defines the support domain for a particle and only those particles within the support domain of the interested particle will contribute to calculate the field variables at that particle.

- The SPH equations for the governing equation of heat conduction is given in Eq. (3).

$$\left(c\frac{\partial T}{\partial t}\right)_i = \sum_{j=1}^{N}\frac{m_j}{\rho_i\rho_j}(k_i+k_j)T_{ij}\frac{1}{r_{ij}}\frac{\partial W_{ij}}{\partial r_{ij}} + \left(\frac{Q_i}{\rho_i}\right) \qquad (3)$$

- The SPH equations for the continuity and momentum are represented in Eqs. (4) and (5) as

$$\frac{\partial \rho_i}{\partial t} = \rho_i \sum_{j=1}^{N}\frac{m_j}{\rho_j}\mathbf{v}_{ij}\nabla_i W_{ij} \qquad (4)$$

$$\frac{d\mathbf{v}_i}{dt} = \sum_{j=1}^{N}\frac{m_j}{\rho_j}\left(\frac{P_i}{\rho_i^2}+\frac{P_j}{\rho_j^2}\right)\nabla_i W_{ij} + \sum_{j=1}^{M} m_j\frac{(\mu_i+\mu_j)\mathbf{v}_{ij}}{\rho_i\rho_j}\left(\frac{1}{r_{ij}}\frac{\partial W_{ij}}{\partial r_i}\right) + F_i \qquad (5)$$

- where ρ, m, v, P and μ are density, mass, velocity, pressure and viscosity of the fluid respectively. $\mathbf{v}_{ij} = \mathbf{v}_j - \mathbf{v}_i$ and F denotes the body force.

Results:

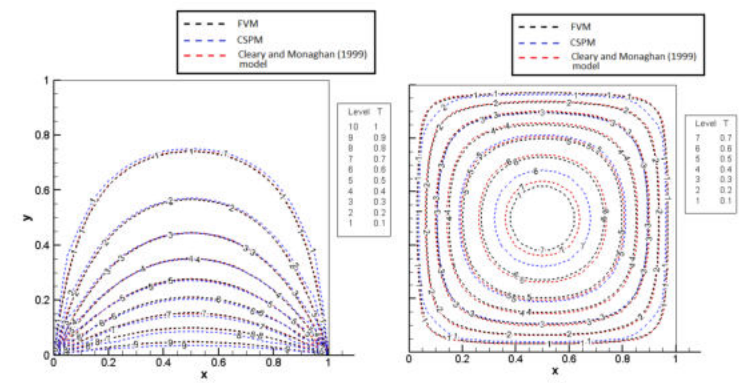

Figure 1. *Comparison of results for two-dimensional heat conduction problem for uniform temperature at bottom(left) and constant heat source(right).*

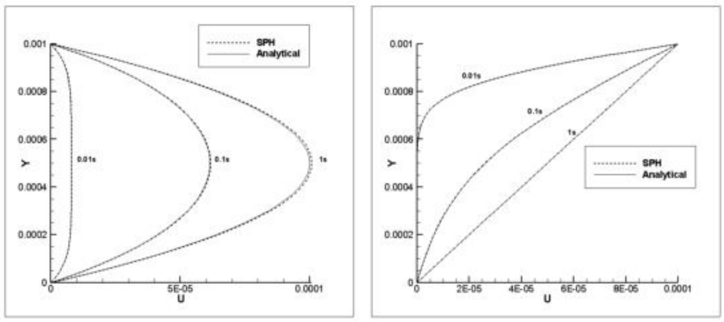

Figure 1. *Comparison of results for Poiseuille flow and Couette flow*

Conclusions:

- The numerical results obtained from the developed computational model are found to be in good agreement with available results in literature.
- In near future, we aim to extend this model to study fluid-structure interaction problems in the field of biofluid dynamics.

Important References:

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